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RETRIAL QUEUEING SYSTEM WITH CATASTROPHE AND IMPATIENT CUSTOMERS

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Abstract. In this paper, a retrial queueing system with catastrophe having impatient customers is studied. If a customer (fresh/primary customer) on arrival finds the server free, it is served immediately, otherwise it joins the virtual queue known as orbit and retries from there after a random amount of time. These retrying customers are known as secondary customers. Two types of impatience are considered.
(i)The primary customers on finding the busy server may leave the system without joining the orbit due to impatience.
(ii)The customers who are retrying from the orbit may get impatient and leave the system without being served.
Moreover, due to some random catastrophic failures, all the present units in the system are deleted and it also causes the break down of the system. Repair of the failed system starts immediately. Primary and secondary customers follow the Poisson process. Catastrophe occurs following the Poisson process. Service times and repair times are distributed exponentially. Time-dependent probabilities for the exact number of arrivals in the system, departures after taking service from the system, and number of customers in orbit when the server is idle or busy are obtained. The probabilities of system being under repair are also obtained. Verification of results is done. Numerical results are generated and represented graphically to study the effect of various parameters.

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1. Introduction

Queueing models play an important role in modeling many real life situations. In queueing literature, we have studied the models with finite and infinite waiting capacity. In models with infinite waiting capacity, customers wait indefinitely before being served. However in today's fast-paced world, people are not willing to wait and they leave the service area forever. There are some situations where the customers on encountering a busy server, instead of leaving the service area forever, tries again after some time to get service. The customers who are retrying for service are known as retrial customers or secondary customers and the associated systems are known as retrial queueing systems. In such systems if a customer does not get service immediately, he temporarily leaves the service area and joins the virtual queue known as orbit and retries for service after a random amount of time. The retrial queueing models are applicable in telecommunication systems,

computer networks, call centers and in data transmission, etc.
A simple example of retrial queueing systems can be seen in call centers where when customers call, if they are able to connect to a call agent, they are answered immediately. Otherwise, they call again after some time.
In recent years many researchers worked on retrial queueing systems. [1] is the work done on retrial queues in its early stages. [2] discussed some important single server retrial queueing models and represented analytic results. [3] analyzed the single server retrial queue with finite number of sources and established customer's arrival distribution, busy period and waiting time process. Since traffic intensity fluctuates in real-world situations, therefore determining transient solutions is crucial for analyzing system behavior. Traditional transient results for the $M/M/1$, $M/M/c$, and $M/G/1$ queues offer limited insights into a queueing system's behavior over a fixed operation time t. While the probability $P_n(t)$ indicates the distribution of the number in the system at time t, it doesn't practically reveal how the system has been regulated up to that point. [4] was the first who introduced the concept of two-state by obtaining a closed form solution for the probability that exactly i arrivals and j services occur over a time interval of length t in 'Some New Results for the M/M/1 Queue'. [5] obtained the time dependent probabilities of exactly i arrivals and j departures by time t for $M/M/1$ queueing model with bernoulli schedule and multiple working vacations. [6] published 'Performance analysis of a two-state queueing model with retrials' in which time dependent probabilities for exact number of arrivals and departures from the system by time t when the server is free or busy are obtained.
An explanation of the retrial queueing system with impatient customers is shown in the following diagram:

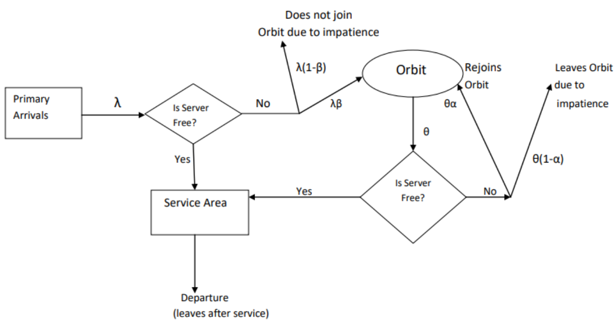

FIGURE 1. Structure of a Retrial Queueing System having impatient customers.

Recently, researchers have focused their attention on catastrophes in retrial queueing models due to their applications in computer and communication systems. The word catastrophe refers to a sudden, unexpected failure of a machine, computer network, electronic system, communication system,

etc. Catastrophes occur randomly, deleting all customers present in the system and inactivating the service facilities. Catastrophe resets the system from current state to failed state at random time intervals. Catastrophe may come from outside the system or from another service station. Retrial queueing models with catastrophe have applications in call centers, computer networks and in telecommunication systems that depend on satellites. Thus the analysis of retrial queueing systems with catastrophe is very important from the application point of view. The loss of customers due to these breakdowns (also referred to as a kind of negative arrivals) was first introduced by [7]. A lot of work on queueing systems with catastrophe can be found in [8] and [9]. [10] discussed mean queue length and the asymptotic behavior of the probability of server being free. Also the steady state probabilities are obtained. [11] worked on M/M/1 queueing system with catastrophe and transient solution for the system with server failure and non-zero repair time is obtained.
A basic example of retrial queueing system with catastrophe is in call centers where if customers are able to reach a live negotiator immediately upon making a call, they are answered else they repeat the call after a couple of minutes. Furthermore, loss of all the customers and inactivation of the server will take place as a result of an incidental power failure or a virus attack.
On the occurrence of catastrophe when the system fails, it is repaired immediately and after getting repaired, it comes back to its working position and becomes ready to accept new customers. Following diagram shows the structure of retrial queueing system with catastrophe.

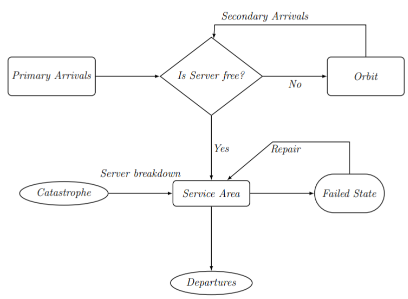

Figure 2. Structure of a Retrial Queue with Catastrophe.

[12] analyzed ‘Computation of the limiting distribution in queueing systems with repeated attempts and disasters’ in which a numerically stable recursion scheme for the state probabilities is derived. Steady state and time dependent solutions for single server retrial queue with catastrophe when the server is idle or busy are obtained in [13].
Retrial queueing systems are also affected by a phenomenon i.e., impatience of customers. Impatience can be of two types:
(i) Sometimes a customer on getting busy server does not want to join the orbit due to impatience and leaves the system without being served.

(ii) While retrying from the orbit if a customer finds busy server again and again then he become impatient and leaves the system without getting service.

In the first case customers do not join the orbit and in the second case customers leave the system after waiting some time for service in the orbit. [14] specifically discussed how to make a decision while waiting in the queue, the probable effect of this decision and the behavior of a queue in which all persons are employing such a procedure. [15] studied multi-server markovian model where the effect of customer's impatience and retrial is evaluated. [16] studied queueing systems with balking and reneging and obtained steady-state solutions. [17] analyzed a single server retrial queueing model with impatient customers. They obtained time dependent probabilities for exact number of arrivals and departures from the system by time t when the server is idle or busy by solving difference-differential equations recursively. [18] obtained time dependent probabilities for two-state feedback retrial queueing system having two heterogeneous servers and impatient customers.

An example of retrial queue with catastrophe and impatient customers can be observed in a bank branch. A bank has several tellers serving customers who arrive to perform various transactions. If a customer on arrival finds the server free, it is served immediately. However, if there is a very long queue then he may decide to leave immediately and not to retry for service due to his impatient behavior. On the other hand, if a customer initially joins the orbit but becomes impatient after making several unsuccessful attempts to get service, he may decide to leave the orbit before being served. In this scenario, a catastrophic event like a system crash, security breaches may occur which forces all customers in the system to be removed.

In this paper, we determine the time-dependent probabilities for the exact number of arrivals in the system, the exact number of departed units after receiving service, and the exact number of customers in the orbit by time t when the server is either idle or busy in a single server retrial queueing system with catastrophes and impatient customers. Additionally, we calculate the probability of the system being under repair when it fails due to random catastrophic events. Alongside these theoretical results, we provide graphical numerical results to examine the behavior of probabilities with respect to average service times. We also conduct sensitivity analysis to investigate the impact of various parameters on the time-dependent probabilities.

The paper has been organized in the following sections:

In section 2, the complete mathematical description of the model is defined. Also, the difference-differential equations are derived in this section. Mathematical analysis of the model is done in section 3 in which we obtained the recursive probabilities. Transient state probabilities and the probability of server being under repair are obtained in section 4. In section 5, verification of results is done. The numerical results are obtained and represented graphically in section 6. Sensitivity analysis is performed in section 7. Section 8 discusses the conclusion. Finally the references are listed.

2. Model Description

In this paper, a single server retrial queueing system with catastrophe having impatient customers is discussed. The primary customers arrive at the system according to Poisson process. On arrival if a customer finds the server free, it is served immediately and departs from the system after getting service. However, if the server is not free then the customer leaves the system without getting service due to impatience with probability $(1-\beta)$ and joins the orbit with probability β and retries from there after a random amount of time. While retrying from the orbit, if a customer finds server free it is served else it may leave the system due to impatience with probability $(1-\alpha)$ and retries again with probability α. Moreover, catastrophe occurs at the system and cause system failure. The failed system is repaired immediately.

Assumptions: The underlying assumptions of the model are given below:

- The primary customers arrive at the system following Poisson process with mean arrival rate λ.
- The retrying customers retry from the orbit following Poisson process with mean retrial rate θ.
- Service times are distributed exponentially with parameter μ.
- Catastrophe occurs at the system following Poisson process with mean rate ξ.
- Repair times follow exponential distribution with parameter τ. Further, it is assumed that during repair time no arrival take place.

Also, the interarrival times, service times, as well as repair times are statistically independent.

Laplace transformation $\bar{f}(s)$ of $f(t)$ is given by:

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)dt; \quad Re(s) > 0$$

$$If \;\; L^{-1}\{\bar{f}(s)\} = F(t) \;\; and \;\; L^{-1}\{\bar{g}(s)\} = G(t) \;\; then$$

$$L^{-1}\{\bar{f}(s)\bar{g}(s)\} = \int_0^t F(u)G(t-u)du = F * G$$

$F * G$ is called the convolution of F and G.

2.1. **Notations.** $P^{(0)}_{i,j,k}(t)$ = Probability that there are exactly i number of arrivals in the system, j number of units departed from the system after taking the service, k number of units in the orbit by time t and the server is free.

$P^{(1)}_{i,j,k}(t)$ = Probability that there are exactly i number of arrivals in the system, j number of units departed from the system after taking the service, k number of units in the orbit by time t and the server is busy.

$Q_{i,j}(t)$ = Probability that there are exactly i number of arrivals, j number of units departed from the system after taking service and the system is under repair by time t.

$P_{i,j,k}(t)$ = Probability that there are exactly i arrivals in the system, j number of units departed from the system after taking the service, k number of units in the orbit by time t.

$$P_{i,j,k}(t) = P^{(0)}_{i,j,k}(t) + P^{(1)}_{i,j,k}(t) \qquad \forall\, i,j,k \;\; i \geq j,k$$
$$and \qquad P^{(1)}_{i,j,k}(t) = 0,\; i \leq j,k; \qquad P^{(0)}_{i,j,k}(t) = 0, \quad i < j,k;$$

Initially

$$P^{(0)}_{0,0,0}(0) = 1; \qquad P^{(0)}_{i,j,k}(0) = 0, \quad P^{(1)}_{i,j,k}(0) = 0,\; i,j,k \neq 0; \qquad Q_{i,j}(0) = 0 \quad \forall\, i,j;$$

2.2. **Difference-Differential Equations.**

(1)

$$\frac{d}{dt}P^{(0)}_{i,j,k}(t) = -(\lambda+\xi+k\theta)P^{(0)}_{i,j,k}(t) + \mu(1-\delta_{0,j})P^{(1)}_{i,j-1,k}(t) + \tau\delta_{0,k}Q_{i,j}(t) \qquad i \geq j, k \geq 0$$

(2)

$$\frac{d}{dt}P^{(1)}_{i,0,k}(t) = -(\lambda\beta+\mu+\xi+k\theta(1-\alpha))P^{(1)}_{i,0,k}(t) + \lambda\delta_{0,k}P^{(0)}_{i-1,0,k}(t) + \lambda\beta (1-\delta_{0,k})P^{(1)}_{i-1,0,k-1}(t) \qquad i \geq 1, k \geq 0$$

(3)

$$\frac{d}{dt}P^{(1)}_{i,j,k}(t) = -(\lambda\beta+\mu+\xi+k\theta(1-\alpha))P^{(1)}_{i,j,k}(t) + \lambda P^{(0)}_{i-1,j,k}(t) + \lambda\beta(1-\delta_{0,k})P^{(1)}_{i-1,j,k-1}(t) + (k+1)\theta P^{(0)}_{i,j,k+1}(t) + (k+1)\theta(1-\alpha)P^{(1)}_{i,j-1,k+1}(t) \qquad i \geq 2, j > 0, k \geq 0$$

(4)

$$\frac{d}{dt}Q_{i,j}(t) = -\tau Q_{i,j}(t) + \xi\left(P^{(0)}_{i,j,0}(t) + \sum_{k=1}^{i-j}(1-\delta_{0,j})P^{(0)}_{i,j,k}(t) + \sum_{k=0}^{i-j-1}P^{(1)}_{i,j,k}(t)\right) \quad i \geq j \geq 0$$

where

$$\delta_{0,j} = \begin{cases} 1 & \text{when } j = 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\delta_{0,k} = \begin{cases} 1 & \text{when } k = 0 \\ 0 & \text{otherwise} \end{cases}$$

3. Mathematical Analysis of the Model

Using the Laplace transformation $\bar{f}(s)$ of $f(t)$ given by

$$\bar{f}(s) = \int_0^\infty e^{-st}f(t)dt, \quad Re(s) > 0$$

in the equations (1)-(4) along with the initial conditions, we have

(5)
$$(s+\lambda+\xi+k\theta)\bar{P}^{(0)}_{i,j,k}(s)=\mu(1-\delta_{0,j})\bar{P}^{(1)}_{i,j-1,k}(s)+\tau\delta_{0,k}\bar{Q}_{i,j}(s) \qquad i\geq j,k\geq 0$$

(6)
$$(s+\lambda\beta+\mu+\xi+k\theta(1-\alpha))\bar{P}^{(1)}_{i,0,k}(s)=\lambda\delta_{0,k}\bar{P}^{(0)}_{i-1,0,k}(s)+\lambda\beta(1-\delta_{0,k})\bar{P}^{(1)}_{i-1,0,k-1}(s) \qquad i\geq 1,k\geq 0$$

(7)
$$(s+\lambda\beta+\mu+\xi+k\theta(1-\alpha))\bar{P}^{(1)}_{i,j,k}(s)=\lambda\bar{P}^{(0)}_{i-1,j,k}(s)+\lambda\beta(1-\delta_{0,k})\bar{P}^{(1)}_{i-1,j,k-1}(s)+(k+1)\theta\,\bar{P}^{(0)}_{i,j,k+1}(s)+(k+1)\theta(1-\alpha)\bar{P}^{(1)}_{i,j-1,k+1}(s) \qquad i\geq 2,j>0,k\geq 0$$

(8)
$$(s+\tau)\bar{Q}_{i,j}(s)=\xi\left(\bar{P}^{(0)}_{i,j,0}(s)+\sum_{k=1}^{i-j}(1-\delta_{0,j})\bar{P}^{(0)}_{i,j,k}(s)+\sum_{k=0}^{i-j-1}\bar{P}^{(1)}_{i,j,k}(s)\right) \qquad i\geq j\geq 0$$

Solving equations (5)-(8) recursively, we have

(9)
$$\bar{Q}_{0,0}(s)=\frac{\xi}{(s+\tau)(s+\lambda+\xi)-\xi\tau}$$

(10)
$$\bar{P}^{(0)}_{0,0,0}(s)=\frac{1}{s+\lambda+\xi}+\frac{\tau}{s+\lambda+\xi}\bar{Q}_{0,0}(s)$$

(11)
$$\bar{P}^{(0)}_{i,0,0}(s)=\frac{\tau}{s+\lambda+\xi}\bar{Q}_{i,0}(s) \qquad i\geq 1$$

(12)
$$\bar{P}^{(0)}_{i,1,0}(s)=\frac{\mu\lambda}{(s+\lambda+\xi)(s+\lambda\beta+\mu+\xi)}\bar{P}^{(0)}_{i-1,0,0}(s)+\frac{\tau}{s+\lambda+\xi}\bar{Q}_{i,1}(s) \qquad i\geq 1$$

(13)
$$\bar{P}^{(0)}_{i,j,0}(s)=\frac{\mu\lambda}{(s+\lambda+\xi)(s+\lambda\beta+\mu+\xi)}\bar{P}^{(0)}_{i-1,j-1,0}(s)+\frac{\mu\theta}{(s+\lambda+\xi)(s+\lambda\beta+\mu+\xi)}\bar{P}^{(0)}_{i,j-1,1}(s)+\frac{\mu\theta(1-\alpha)}{(s+\lambda+\xi)(s+\lambda\beta+\mu+\xi)}\bar{P}^{(1)}_{i,j-2,1}(s)+\frac{\tau}{s+\lambda+\xi}\bar{Q}_{i,j}(s) \qquad i\geq j\geq 2$$

(14)
$$\bar{P}^{(0)}_{i,1,k}(s)=\frac{\mu}{s+\lambda+\xi+k\theta}\left[\frac{\lambda\beta}{s+\lambda\beta+\mu+\xi+k\theta(1-\alpha)}\bar{P}^{(1)}_{i-1,0,k-1}(s)\right] \qquad i\geq k+1,k\geq 1$$

(15)
$$\bar{P}^{(1)}_{i,0,0}(s)=\frac{\lambda}{s+\lambda\beta+\mu+\xi}\bar{P}^{(0)}_{i-1,0,0}(s) \qquad i\geq 1$$

(16)
$$\bar{P}^{(1)}_{i,0,k}(s)=\frac{\lambda\beta}{(s+\lambda\beta+\mu+\xi+k\theta(1-\alpha))}\bar{P}^{(1)}_{i-1,0,k-1}(s) \qquad i>k\geq 1$$

$$\bar{P}^{(0)}_{i,j,k}(s) = \frac{\mu}{s+\lambda+\xi+k\theta}\Bigg[\sum_{p=0}^{k+1}\Bigg\{\prod_{m=p}^{k}\left(\frac{1}{s+\lambda\beta+\mu+\xi+m\theta(1-\alpha)}\right)^{\psi'_p(s)}(\lambda)^{\psi'_p(s)}\Bigg\}$$

$$(\lambda\beta)^{(k-p)\psi'_p(s)}\eta'_p(s)\bar{P}^{(0)}_{i-k-1+p,j-1,p}(s) + \sum_{p=0}^{k}(\lambda\beta)^{k-p}(p+1)\theta(1-\alpha)$$

(17)

$$\Bigg\{\prod_{m=p}^{k}\frac{1}{s+\lambda\beta+\mu+\xi+m\theta(1-\alpha)}\Bigg\}\bar{P}^{(1)}_{i-k+p,j-2,p+1}(s)\Bigg] \qquad i \geq k+j, j > k \geq 1$$

where

$$\psi'_p = \begin{cases} 1 & \text{if } p = 0 \text{ to } k \\ 0 & \text{if } p = k+1 \end{cases}$$

$$\eta'_p = \begin{cases} 1 & \text{if } p = 0 \\ 1 + \dfrac{p\theta\beta}{s+\lambda\beta+\mu+\xi+(p-1)\theta(1-\alpha)} & \text{if } p = 1 \; to \; k \\ \dfrac{p\theta}{s+\lambda\beta+\mu+\xi+(p-1)\theta(1-\alpha)} & \text{if } p = k+1 \end{cases}$$

$$\bar{P}^{(1)}_{i,j,k}(s) = \sum_{p=0}^{k+1}\Bigg\{\prod_{m=p}^{k}\left(\frac{1}{s+\lambda\beta+\mu+\xi+m\theta(1-\alpha)}\right)^{\psi'_p(s)}(\lambda)^{\psi'_p(s)}\Bigg\}$$

$$(\lambda\beta)^{(k-p)\psi'_p(s)}\eta'_p(s)\bar{P}^{(0)}_{i-k-1+p,j,p}(s) + \sum_{p=0}^{k}(\lambda\beta)^{k-p}(p+1)\theta(1-\alpha)$$

(18)

$$\Bigg\{\prod_{m=p}^{k}\frac{1}{s+\lambda\beta+\mu+\xi+m\theta(1-\alpha)}\Bigg\}\bar{P}^{(1)}_{i-k+p,j-1,p+1}(s) \qquad i \geq j+k+1, \quad j,k \geq 1$$

where

$$\eta'_p = \begin{cases} 1 & \text{if } p = 0 \\ 1 + \dfrac{p\theta\beta}{s+\lambda\beta+\mu+\xi+(p-1)\theta(1-\alpha)} & \text{if } p = 1 \; to \; k \\ \dfrac{p\theta}{s+\lambda\beta+\mu+\xi+(p-1)\theta(1-\alpha)} & \text{if } p = k+1 \end{cases}$$

$$P^{(1)}_{i,j,0}(s) = \frac{\lambda}{s+\lambda\beta+\mu+\xi}\bar{P}^{(0)}_{i-1,j,0}(s) + \frac{\theta}{s+\lambda\beta+\mu+\xi}\bar{P}^{(0)}_{i,j,1}(s)$$

(19) $$+\frac{\theta(1-\alpha)}{s+\lambda\beta+\mu+\xi}\bar{P}^{(1)}_{i,j-1,1}(s) \qquad i > j \geq 1$$

(20)
$$\bar{Q}_{i,0}(s) = \frac{\xi}{s+\tau}\left[\bar{P}^{(0)}_{i,0,0}(s) + \sum_{k=0}^{i-1}\bar{P}^{(1)}_{i,0,k}(s)\right] \qquad i \geq 1$$

(21)
$$\bar{Q}_{i,j}(s) = \frac{\xi}{s+\tau}\left[\sum_{k=0}^{i-j}\bar{P}^{(0)}_{i,j,k}(s) + \sum_{k=0}^{i-j-1}\bar{P}^{(1)}_{i,j,k}(s)\right] \qquad i \geq j \geq 1$$

4. Transient Solution of the Model

Taking the Inverse Laplace transform of equations $(9)-(21)$, we obtained the following transient state probabilities.

$$Q_{0,0}(t) = 2\xi\left[\left(\lambda^2+\tau^2+\xi^2+2\left(\tau\xi+\lambda\xi-\lambda\tau\right)\right)^{1/2}e^{-1/2(\lambda+\xi+\tau)t}\right.$$

(22)
$$\left.\sinh\left(\frac{1}{2}t\left\{\lambda^2+2\lambda(\xi-\tau)+(\xi+\tau)^2\right\}^{1/2}\right)\right]^{-1}$$

(23)
$$P^{(0)}_{0,0,0}(t) = e^{-(\lambda+\xi)t} + \tau e^{-(\lambda+\xi)t} * Q_{0,0}(t)$$

(24)
$$P^{(0)}_{i,0,0}(t) = \tau e^{-(\lambda+\xi)t} * Q_{i,0}(t) \qquad i \geq 1$$

$$P^{(0)}_{i,1,0}(t) = \mu\lambda e^{-(\lambda+\xi)t}\left\{\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)} - \frac{e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)t}}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)}\right\} * P^{(0)}_{i-1,0,0}(t) + \tau e^{-(\lambda+\xi)t}$$

(25)
$$* Q_{i,1}(t) \qquad i \geq 1$$

(26)
$$P^{(1)}_{i,0,0}(t) = \lambda e^{-(\lambda\beta+\mu+\xi)t} * P^{(0)}_{i-1,0,0}(t) \qquad i \geq 1$$

$$P^{(0)}_{i,1,k}(t) = \mu(\lambda\beta)e^{-(\lambda+\xi+k\theta)t}\left\{\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{k\theta(1-\alpha)}{\beta}\right)} - \frac{e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{k\theta(1-\alpha)t}{\beta}\right)}}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{k\theta(1-\alpha)}{\beta}\right)}\right\}$$

(27)
$$* P^{(1)}_{i-1,0,k-1}(t) \qquad i \geq k+1, k \geq 1$$

$$P^{(0)}_{i,j,0}(t)=\mu\lambda e^{-(\lambda+\xi)t}\left\{\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)}-\frac{e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)t}}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)}\right\}*P^{(0)}_{i-1,j-1,0}(t)+\mu\theta e^{-(\lambda+\xi)t}$$

$$\left\{\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)}-\frac{e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)t}}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)}\right\}*P^{(0)}_{i,j-1,1}(t)+\mu\theta(1-\alpha)e^{-(\lambda+\xi)t}$$

(28)

$$\left\{\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)}-\frac{e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)t}}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}\right)}\right\}*P^{(1)}_{i,j-2,1}(t)+\tau e^{-(\lambda+\xi)t}*Q_{i,j}(t) \qquad i\geq j\geq 2$$

(29)

$$P^{(1)}_{i,0,k}(t)=(\lambda\beta)e^{-(\lambda\beta+\mu+\xi+k\theta(1-\alpha))t}*P^{(1)}_{i-1,0,k-1}(t) \qquad i>k\geq 1$$

$$P^{(1)}_{i,j,0}(t)=\lambda e^{-(\lambda\beta+\mu+\xi)t}*P^{(0)}_{i-1,j,0}(t)+\theta e^{-(\lambda\beta+\mu+\xi)t}*P^{(0)}_{i,j,1}(t)+\theta(1-\alpha)$$

(30)

$$e^{-(\lambda\beta+\mu+\xi)t}*P^{(1)}_{i,j-1,1}(t) \qquad i>j\geq 1$$

(31)

$$Q_{i,0}(t)=\xi e^{-\tau t}*\left[P^{(0)}_{i,0,0}(t)+\sum_{k=0}^{i-1}P^{(1)}_{i,0,k}(t)\right] \qquad i\geq 1$$

(32)

$$Q_{i,j}(t)=\xi e^{-\tau t}*\left[\sum_{k=0}^{i-j}P^{(0)}_{i,j,k}(t)+\sum_{k=0}^{i-j-1}P^{(1)}_{i,j,k}(t)\right] \qquad i\geq j\geq 1$$

$$P^{(0)}_{i,j,k}(t)=\mu e^{-(\lambda+\xi+k\theta)t}(\lambda)(\lambda\beta)^k\left\{\prod_{m=0}^{k}\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m+1}}-e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)t}\right.$$

$$\left.\sum_{r=0}^{m}\frac{t^r}{r!}\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-r+1}}\right\}*P^{(0)}_{i-k-1,j-1,0}(t)+\lambda\mu e^{-(\lambda+\xi+k\theta)t}\left[\sum_{p=1}^{k}(\lambda\beta)^{k-p}\right.$$

$$\left\{\prod_{m=p}^{k}\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p+1}}-e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)t}\sum_{r=0}^{m-p}\frac{t^r}{r!}\right.$$

$$\left.\left.\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p-r+1}}\right\}*P^{(0)}_{i-k-1+p,j-1,p}(t)\right]+\lambda\mu e^{-(\lambda+\xi+k\theta)t}\left[\sum_{p=1}^{k}(\lambda\beta)^{k-p}(p\theta\beta)\right.$$

$$\left\{\prod_{m=p-1}^{k}\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p+2}}-e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)t}\sum_{r=0}^{m-p+1}\frac{t^r}{r!}\right.$$

$$\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p-r+2}}\Bigg\}*P^{(0)}_{i-k-1+p,j-1,p}(t)\Bigg]+\mu(k+1)\theta e^{-(\lambda+\xi+k\theta)t}$$

$$\left\{\frac{1}{\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{k\theta(1-\alpha)}{\beta}}-\frac{e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{k\theta(1-\alpha)}{\beta}\right)t}}{\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{k\theta(1-\alpha)t}{\beta}}\right\}*P_{i,j-1,k+1}(t)+\mu e^{-(\lambda+\xi+k\theta)t}$$

$$\Bigg[\sum_{p=0}^{k}(\lambda\beta)^{k-p}(p+1)\theta(1-\alpha)\Bigg\{\prod_{m=p}^{k}\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p+1}}$$

$$-e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)t}\sum_{r=0}^{m-p}\frac{t^r}{r!}\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p-r+1}}\Bigg\}*P^{(1)}_{i-k+p,j-2,p+1}(t)\Bigg]$$

(33)

$$i\geq k+j+1,\, j>k\geq 1$$

$$P^{(1)}_{i,j,k}(t)=(\lambda)(\lambda\beta)^k\Bigg\{\prod_{m=0}^{k}\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m+1}}-e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)t}$$

$$\sum_{r=0}^{m}\frac{t^r}{r!}\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-r+1}}\Bigg\}*P^{(0)}_{i-k-1,j,0}(t)+\lambda\sum_{p=1}^{k}(\lambda\beta)^{k-p}$$

$$\Bigg\{\prod_{m=p}^{k}\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p+1}}-e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)t}\sum_{r=0}^{m-p}\frac{t^r}{r!}$$

$$\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p-r+1}}\Bigg\}*P^{(0)}_{i-k-1+p,j,p}(t)+\sum_{p=1}^{k}(\lambda\beta)^{k-p}(p\theta\beta)\Bigg\{\prod_{m=p-1}^{k}$$

$$\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p+2}}-e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)t}\sum_{r=0}^{m-p+1}\frac{t^r}{r!}$$

$$\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p-r+2}}\Bigg\}*P^{(0)}_{i-k-1+p,j,p}(t)+(k+1)\theta e^{-(\lambda\beta+\mu+\xi+k\theta(1-\alpha))t}$$

$$*P^{(0)}_{i,j,k+1}(t)+\sum_{p=0}^{k}(\lambda\beta)^{k-p}(p+1)\theta(1-\alpha)\Bigg\{\prod_{m=p}^{k}\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p+1}}$$

$$-e^{-\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)t}\sum_{r=0}^{m-p}\frac{t^r}{r!}\frac{1}{\left(\frac{\mu}{\beta}+\frac{\xi}{\beta}+\frac{m\theta(1-\alpha)}{\beta}\right)^{m-p-r+1}}\Bigg\}*P^{(1)}_{i-k+p,j-1,p+1}(t)$$

(34)
$$i \geq j+k+1, j, k \geq 1$$

5. Verification of Results

- Taking $\xi = 0$, $\tau = 0$ and

$P^{(0)}_{i,j,k}(t) = P^{(0)}_{i,j}(t)$ where $k =$ number of arrivals - number of departures.
$P^{(1)}_{i,j,k}(t) = P^{(1)}_{i,j}(t)$ where $k =$ number of arrivals - number of departures - 1.

in equations $(1-4)$ we get,

(35)
$$\frac{d}{dt}P^{(0)}_{i,j}(t) = -(\lambda + (i-j)\theta)P^{(0)}_{i,j}(t) + \mu(1-\delta_{0,j})P^{(1)}_{i,j-1}(t) \qquad i \geq j \geq 0$$

(36)
$$\frac{d}{dt}P^{(1)}_{1,0}(t) = -(\lambda\beta + \mu)P^{(1)}_{i,0}(t) + \lambda P^{(0)}_{0,0}(t)$$

$$\frac{d}{dt}P^{(1)}_{i,j}(t) = -(\lambda\beta + \mu + (i-j-1)\theta(1-\alpha))P^{(1)}_{i,j}(t) + \lambda P^{(0)}_{i-1,j}(t)$$
$$+ \lambda\beta(1-\delta_{i-1,j})P^{(1)}_{i-1,j}(t) + (i-j)\theta P^{(0)}_{i,j}(t) + (i-j)\theta(1-\alpha)(1-\delta_{0,j})P^{(1)}_{i,j-1}(t)$$

(37)
$$i > 1, i > j \geq 0$$

where
$$\delta_{i-1,j} = \begin{cases} 1 & \text{when } i-1=j \\ 0 & \text{otherwise} \end{cases}$$

which coincides with the results (1-3) of [17].

Furthermore, the model given in equations $(35)-(37)$ can be converted into one-dimensional model by defining the probability $U^{(m)}_n(t)$ as:

$U^{(m)}_n(t) =$ Probability that there are n customers in the orbit at time t and the server is free or busy according as $m = 0$ *or* 1.

when server is free, it is defined as probability:

$$U^{(0)}_n(t) = \sum_{j=0}^{\infty} P^{(0)}_{j+n,j}(t)$$

where n = number of arrivals - number of departures.
when server is busy, it is defined as probability:

$$U^{(1)}_n(t) = \sum_{j=0}^{\infty} P^{(1)}_{j+n+1,j}(t)$$

where n = number of arrivals - number of departures - 1.
By using above definitions in equations $(35-37)$ the equations in

statistical equilibrium are:

$$(\lambda + n\theta)U_n^{(0)} = \mu U_n^{(1)} \qquad n \geq 0$$

$$(\lambda\beta + \mu + n\theta(1-\alpha))U_n^{(1)} = \lambda U_n^{(0)} + \lambda\beta U_{n-1}^{(1)} + (n+1)\theta U_{n+1}^{(0)} + (n+1)\theta(1-\alpha)U_{n+1}^{(1)} \quad n \geq 1$$

which coincides with the results (3.68) of [19]

6. Numerical Solution and Graphical Representation

Following numerical results are evaluated using MATLAB programming for the case $\rho = \left(\frac{\lambda}{\mu}\right)$=0.7, $\eta = \left(\frac{\theta}{\mu}\right)$=0.6, $\tau' = \left(\frac{\tau}{\mu}\right)$=0.4, $\xi' = \left(\frac{\xi}{\mu}\right)$=0.3, $\alpha = 0.5$, $\beta = 0.8$. By taking a look at the tables below for various time instants t, it can be seen that the sum of probabilities approaches to 1.

Table 1. At time t =1

t	$P_{0,0,0}^{(0)}$	$P_{1,1,0}^{(0)}$	$P_{2,1,0}^{(0)}$	$P_{2,1,1}^{(0)}$	$P_{2,2,0}^{(0)}$	$P_{1,0,0}^{(1)}$	$P_{2,0,0}^{(1)}$	$P_{2,0,1}^{(1)}$	$P_{3,0,2}^{(1)}$	$Q_{0,0}$	$Q_{1,0}$	$Q_{1,1}$	Sum
1	0.3954	0.1011	0.0005	0.0128	0.0069	0.1777	0.0007	0.0385	0.0059	0.1542	0.0379	0.0126	0.9479

Table 2. At time t =30

t	$P_{5,1,0}^{(0)}$	$P_{5,2,0}^{(0)}$	$P_{5,3,0}^{(0)}$	$P_{5,4,0}^{(0)}$	$P_{5,5,0}^{(0)}$	$Q_{5,1}$	$Q_{5,2}$	$Q_{5,3}$	$Q_{5,4}$	$Q_{5,5}$	Sum
30	0.0366	0.1036	0.1674	0.1591	0.0751	0.0278	0.078	0.1248	0.1175	0.0552	0.9451

Table 3. At time t =35

t	$P_{5,1,0}^{(0)}$	$P_{5,2,0}^{(0)}$	$P_{5,3,0}^{(0)}$	$P_{5,4,0}^{(0)}$	$P_{5,5,0}^{(0)}$	$Q_{5,1}$	$Q_{5,2}$	$Q_{5,3}$	$Q_{5,4}$	$Q_{5,5}$	Sum
35	0.0376	0.1065	0.1719	0.1628	0.0764	0.0283	0.08	0.1285	0.1212	0.0568	0.97

Table 4. At time t =40

t	$P_{5,1,0}^{(0)}$	$P_{5,2,0}^{(0)}$	$P_{5,3,0}^{(0)}$	$P_{5,4,0}^{(0)}$	$P_{5,5,0}^{(0)}$	$Q_{5,1}$	$Q_{5,2}$	$Q_{5,3}$	$Q_{5,4}$	$Q_{5,5}$	Sum
40	0.038	0.1079	0.1739	0.1644	0.077	0.0286	0.0809	0.1303	0.1229	0.0575	0.9814

The probabilities against time are represented graphically in the following figures:

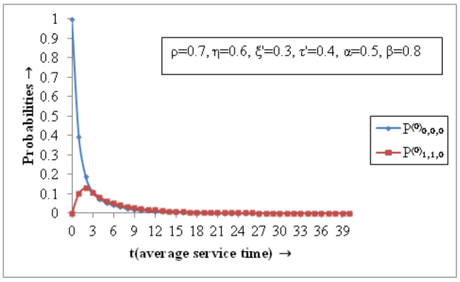

FIGURE 3. Probabilities $P^{(0)}_{0,0,0}$ and $P^{(0)}_{1,1,0}$ against average service times t.

In figure 3, probabilities $P^{(0)}_{0,0,0}$ and $P^{(0)}_{1,1,0}$ are plotted against times t. It can be observed from the figure that the probability $P^{(0)}_{0,0,0}$ decreases rapidly from its initial value 1 at $t = 0$ whereas the probability $P^{(0)}_{1,1,0}$ increases from its initial value 0 at $t = 0$ and then decreases gradually.

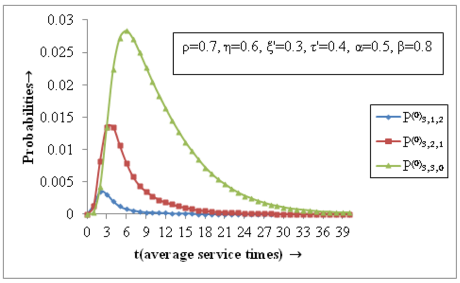

FIGURE 4. Probabilities $P^{(0)}_{3,1,2}$, $P^{(0)}_{3,2,1}$ and $P^{(0)}_{3,3,0}$ against average service times t.

Figure 4 shows the comparison among the probabilities $P^{(0)}_{3,1,2}$, $P^{(0)}_{3,2,1}$ and $P^{(0)}_{3,3,0}$ against average service times t. Here we observe that all probabilities increase in the starting time points and then decrease after reaching their peak. Moreover, the probabilities attain higher values for greater number of departures i.e., more departures, more will be the probability of server being free.

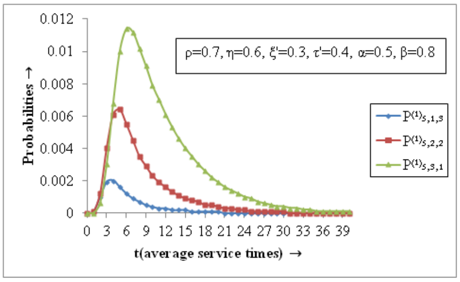

Figure 5. Probabilities $P^{(1)}_{5,1,3}$, $P^{(1)}_{5,2,2}$ and $P^{(1)}_{5,3,1}$ against average service times t.

In figure 5, the probabilities $P^{(1)}_{5,1,3}$, $P^{(1)}_{5,2,2}$ and $P^{(1)}_{5,3,1}$ are plotted against average service times t. It is clearly visible from the graph that the probabilities increase initially and then start decreasing with time. Also the probability of server being busy is highest for larger number of departures when the arrivals remain constant.

7. Sensitivity Analysis of the Model

Under this section, the effect of change in parameters of the system on the various probabilities is studied. The data is summarized in the following tables. In table 5, the probability $P^{(0)}_{0,0,0}$ is given for different values of ρ. It can be seen that in general whenever the arrival rate per unit service time increases the probability $P^{(0)}_{0,0,0}$ decreases against average service times t. In table 6, the effect of catastrophe rate per unit service time on the probability of system being under repair is studied. It is observed that the probability of system being under repair increases with the increase in ξ'. Similarly, the effect of repair rate per unit service time on the probability of system being under repair is studied in table 7. Here we notice that the probability decreases with the increase in τ'.

Table 5. Effect of ρ on Probabilities

	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.7$
t	$P^{(0)}_{0,0,0}$	$P^{(0)}_{0,0,0}$	$P^{(0)}_{0,0,0}$
0	1	1	1
3	0.2861	0.1745	0.1092
6	0.1602	0.0772	0.0413
9	0.1033	0.0414	0.0196
12	0.0678	0.0227	0.0095
15	0.0446	0.0124	0.0046
18	0.0294	0.0068	0.0022
21	0.0193	0.0037	0.0011
24	0.0127	0.0021	0.0005
27	0.0084	0.0011	0.0003
30	0.0055	0.0006	0.0001
33	0.0036	0.0003	0.0001
36	0.0024	0.0002	0
39	0.0016	0.0001	0

Table 6. Effect of ξ' on probabilities

	$\xi' = 0.3$			$\xi' = 0.6$			$\xi' = 0.9$		
t	$Q_{2,1}$	$Q_{3,1}$	$Q_{4,1}$	$Q_{2,1}$	$Q_{3,1}$	$Q_{4,1}$	$Q_{2,1}$	$Q_{3,1}$	$Q_{4,1}$
1	0.0034	0.0005	0.0001	0.0054	0.0008	0.0001	0.0066	0.001	0.0001
3	0.0326	0.0118	0.0034	0.0417	0.0146	0.0041	0.0416	0.014	0.0037
6	0.0446	0.0239	0.0105	0.0611	0.0317	0.0131	0.0641	0.0318	0.0124
9	0.0365	0.0243	0.013	0.0606	0.0395	0.0202	0.0708	0.0441	0.0213
12	0.0261	0.0204	0.0126	0.0527	0.0406	0.0242	0.0685	0.0505	0.0286
15	0.0174	0.0154	0.0107	0.0424	0.0374	0.0253	0.0612	0.0519	0.0334
18	0.0111	0.0109	0.0084	0.0324	0.0321	0.0241	0.0519	0.0496	0.0356
21	0.0068	0.0074	0.0061	0.0239	0.0261	0.0216	0.0424	0.0449	0.0355
24	0.0041	0.0048	0.0043	0.0171	0.0205	0.0183	0.0336	0.039	0.0336
27	0.0024	0.003	0.0029	0.0119	0.0155	0.015	0.026	0.0329	0.0306
30	0.0014	0.0019	0.0019	0.0082	0.0115	0.0119	0.0198	0.027	0.027
33	0.0008	0.0011	0.0012	0.0055	0.0083	0.0092	0.0148	0.0217	0.0232
36	0.0004	0.0007	0.0008	0.0037	0.0059	0.007	0.011	0.0172	0.0195
39	0.0002	0.0004	0.0005	0.0024	0.0042	0.0052	0.008	0.0134	0.0161

TABLE 7. Effect of τ' on probabilities

	$\tau' = 0.2$			$\tau' = 0.4$			$\tau' = 0.9$		
t	$Q_{3,0}$	$Q_{3,1}$	$Q_{3,2}$	$Q_{3,0}$	$Q_{3,1}$	$Q_{3,2}$	$Q_{3,0}$	$Q_{3,1}$	$Q_{3,2}$
1	0.0007	0.0005	0.0001	0.0007	0.0005	0.0001	0.0007	0.0005	0.0001
3	0.0064	0.0128	0.0085	0.0059	0.0118	0.008	0.0048	0.0099	0.0067
6	0.0094	0.0268	0.0296	0.0083	0.0239	0.0263	0.0057	0.0175	0.0197
9	0.0097	0.0296	0.0358	0.0077	0.0243	0.0297	0.0039	0.0138	0.018
12	0.0093	0.0289	0.0355	0.0062	0.0204	0.0258	0.0021	0.0081	0.0114
15	0.0084	0.0267	0.0329	0.0045	0.0154	0.0199	0.001	0.0041	0.006
18	0.0074	0.0237	0.0292	0.0031	0.0109	0.0142	0.0004	0.0019	0.0028
21	0.0063	0.0204	0.0251	0.0021	0.0074	0.0096	0.0002	0.0008	0.0012
24	0.0053	0.0171	0.0211	0.0013	0.0048	0.0063	0.0001	0.0003	0.0005
27	0.0043	0.0141	0.0174	0.0008	0.003	0.004	0	0.0001	0.0002
30	0.0035	0.0115	0.0141	0.0005	0.0019	0.0025	0	0	0.0001
33	0.0028	0.0092	0.0113	0.0003	0.0011	0.0015	0	0	0
36	0.0022	0.0073	0.009	0.0002	0.0007	0.0009	0	0	0
39	0.0017	0.0057	0.007	0.0001	0.0004	0.0005	0	0	0

This can also be shown graphically in following figures:

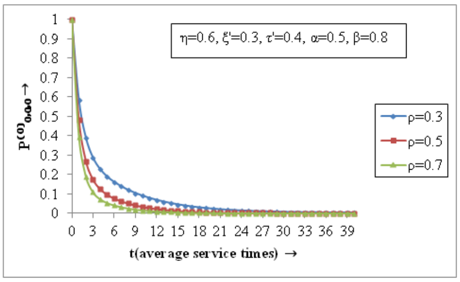

FIGURE 6. Probability $P^{(0)}_{0,0,0}$ for different values of ρ against average service times t.

In figure 6, the probability $P^{(0)}_{0,0,0}$ is plotted for different values of ρ against average service times t. It can be seen that whenever the value of ρ increases, the probability of having zero customers in the system decreases.

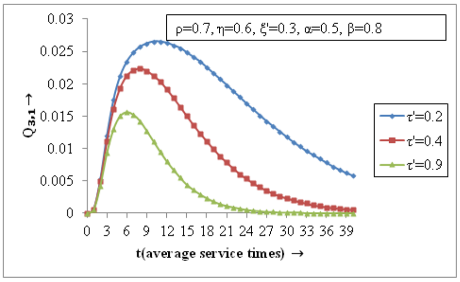

FIGURE 7. Probability of system being under repair for different values of $\xi^{'}$.

In figure 7, the probability of system being under repair is shown when the number of arrivals is three and number of services is one for different values of $\xi^{'}$ (catastrophe rate per unit service time). It is observed from the graph that higher the value of $\xi^{'}$, higher is the probability of system being under repair.

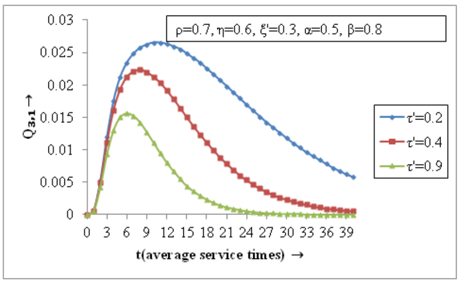

FIGURE 8. Effect of $\tau^{'}$ on the probability $Q_{3,1}$.

In figure 8, the effect of $\tau^{'}$ (change of repair rate per unit service time) on the probability $Q_{3,1}$ is studied. It is observed from the graph that the probability of system being under repair decreases with the increasing values of repair rate per unit service time.

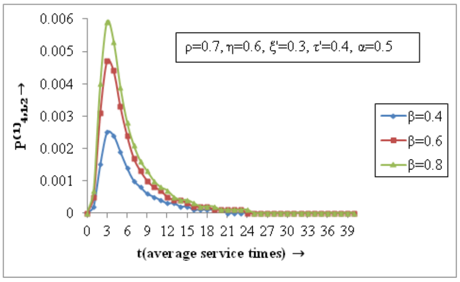

FIGURE 9. Probability $P^{(1)}_{4,1,2}$ for different values of β.

In figure 9, the probability $P^{(1)}_{4,1,2}$ is plotted against average service times t for different values of β. It can be seen that the shape of the curve is increasing in the beginning and then is decreasing after reaching the peak. It is also observed that if the value of β increases, i.e., the probability with which customers join the orbit on finding the busy server increases, the probability $P^{(1)}_{4,1,2}$ also increases.

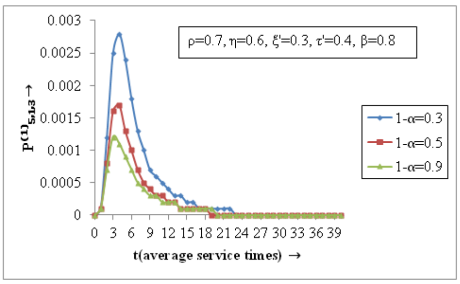

FIGURE 10. Probability $P^{(1)}_{5,1,3}$ for different values of $1-\alpha$.

In figure 10, the probability $P^{(1)}_{5,1,3}$ is plotted against average service times t for different values of $1-\alpha$. It can be seen that the probability increases rapidly in the beginning and then decreases gradually. Moreover, it is observed that if the probability with which customers leave the orbit due to impatience increases, the probability $P^{(1)}_{5,1,3}$ decreases.

8. Conclusion

In queueing systems, customer impatience can adversely affect customer satisfaction, operational efficiency, and overall quality of service. Many businesses and organizations that deal with queues, such as retail stores, banks, restaurants, transportation systems, and others, must understand and manage customer impatience. Incorporating strategies to address customer impatience in queueing systems can lead to higher customer satisfaction, improved operational efficiency, and a more positive overall customer experience. In this paper, we analyzed a single server retrial queueing system with catastrophe having impatient customers. Here we obtained transient solutions for exact number of arrivals in the system, exact number of departed units after taking service from the system and exact number of customers in the orbit by time t when the server is idle or busy. Furthermore, numerical results are generated using MATLAB programming and represented graphically. Sensitivity analysis is performed to study the effect of various parameters on the transient probabilities.

References

[1] Cohen, JW, *Basic problems of telephone traffic theory and the influence of repeated calls*, Philips Telecommunication Review. 18(2), (1957), 49-100.

[2] Yang, Tao and Templeton, James G. C., *A survey on retrial queues*, Queueing systems. 2(3), (1987), 201-233.

[3] Falin, Gennadi I and Artalejo, Jesus R, *A finite source retrial queue*, European Journal of Operational Research. 108(2), (1998), 409-424.

[4] Pegden, Claude Dennis and Rosenshine, Matthew, *Some new results for the M/M/1 queue*, Management Science. 28(7), (1982), 821-828.

[5] Indra, Renu, *Transient analysis of markovian queueing model with bernoulli schedule and multiple working vacations*, Int. J. Comput. Appl. 20, (2011), 43-48.

[6] Singla, Neelam and Kalra, Sonia, *Performance analysis of a two-state queueing model with retrials*, Journal of Rajasthan Academy of Physical Sciences. 17, (2018), 81-100.

[7] Gelenbe, Erol, *Product-form queueing networks with negative and positive customers*, Journal of applied probability. 28(3), (1991), 656-663.

[8] Chao, Xiuli, *A queueing network model with catastrophes and product form solution*, Operations Research Letters. 18(2), (1995), 75-79.

[9] Di Crescenzo, Antonio and Giorno, Virginia and Nobile, Amelia G and Ricciardi, Luigi M, *On the M/M/1 queue with catastrophes and its continuous approximation*, Queueing Systems, 43(4), (2003), 329-347.

[10] Kumar, B Krishna and Arivudainambi, D, *Transient solution of an M/M/1 queue with catastrophes*, Computers & Mathematics with applications, 40(10-11), (2000), 1233-1240.

[11] Kumar, B Krishna and Krishnamoorthy, Achyutha and Madheswari, S Pavai and Basha, S Sadiq, *Transient analysis of a single server queue with catastrophes, failures and repairs*, Queueing systems, 56(3), (2007), 133-141.

[12] Artalejo, Jesus R and Gomez-Corral, Antonio, *Computation of the limiting distribution in queueing systems with repeated attempts and disasters*, RAIRO-Operations Research, 33(3), (1999), 371-382.

[13] Singla, Neelam and Garg, Ankita, *Single Server Retrial Queueing System with Catastrophe*, Reliability: Theory & Applications, 17, 3(69), (2022), 376-390.

[14] Haight, Frank A, *Queueing with reneging*, Metrika, 2(1), (1959), 186-197.

[15] Artalejo, Jesus R and Pla, Vicent, *On the impact of customer balking, impatience and retrials in telecommunication systems*, Computers & Mathematics with Applications, 57(2), (2009), 217-229.

[16] Ancker Jr, CJ and Gafarian, Antranig V, *Some queuing problems with balking and reneging. I*, Operations Research, 11(1), (1963), 88-100.

[17] Singla, Neelam and Kalra, Sonia, *AN ANALYSIS OF A TWO-STATE RETRIAL QUEUE WITH IMPATIENT CUSTOMERS.*, International Journal of Agricultural & Statistical Sciences, 15(2), (2019), 861-871.

[18] Singla, Neelam and Kaur, Harwinder, *A TWO-STATE FEEDBACK RETRIAL QUEUEING SYSTEM HAVING TWO HETEROGENEOUS PARALLEL SERVERS AND IMPATIENT CUSTOMERS*, Reliability: Theory & Applications, 18, 2(73), (2023), 498-512.
[19] Falin, Gennadi and Templeton, James GC, *Retrial queues*, 75, (1997).

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